Soln for [09-01-21-NEM11] #11

Solve for x and y if $64\,(4^y) = 16^x$ and $3^y = 4\,(3^{x-2}) - 1$.

(EQ1) $64\,(4^y) = 16^x \Longleftrightarrow 2^6 \cdot 2^{2\,y} = 2^{4\,x} \Longleftrightarrow 2^{2\,y+6} = 2^{4\,x} \Longrightarrow 2\,y + 6 = 4\,x \Longleftrightarrow\ y = 2\,x - 3$ (EQ1').

Substituting into the second equation (EQ2).

$3^y = 4\,(3^{x-2}) - 1$

$\Longleftrightarrow 3^{2\,x-3} = 4\,(3^{x-2}) - 1$

$\Longleftrightarrow 3^{2\,x} = 4\,(3^{x+1}) - 3^3$, muliplying both sides previous eqn by 3^3.

$\Longleftrightarrow 3^{2\,x} = 12\,(3^x) - 27$

$\Longleftrightarrow (3^x)^2 = 12\,(3^x) - 27$, this is a quadratic equation which following substitution makes obvious

Let $u = 3^x$. Then,

$\Longleftrightarrow u^2 = 12\,u - 27$

$\Longleftrightarrow u^2 - 12\,u + 27 = 0$

$\Longleftrightarrow (u - 9)\,(u - 3) = 0$

$\Longrightarrow u = 9 \lor u = 3$

For $u = 9$, $3^x = 9 \Longrightarrow x = 2$.

When $x = 2$, substitute into (EQ1') $y = 2\,(2) - 3 = 1$

For $u = 3$, $3^x = 3 \Longrightarrow x = 1$.

When $x = 1$, substitute into (EQ1') $y = 2\,(1) - 3 = -1$

Therefore, there appear to be two solutions,

$x = 2,\ y = 1$ or $x = 1,\ y = -1$.

Since substituting these pairs of values into the original equations produces true statements, each pair solves the original system of equations.